Indian Statistical Institute, Bangalore
M. Math II-JRF Math I
First Semester - Topology III

Mid-Term Exam   Duration: 3 hours   Date : Sep 09, 2016

Max Marks: 30

(1) (a) Show that every map $f : (\mathbb{D}^k, \mathbb{S}^{k-1}) \to (\mathbb{D}^{n+1}, \mathbb{S}^n)$ is homotopic relative to $\mathbb{S}^{k-1}$ to a map $g : \mathbb{D}^k \mapsto \mathbb{S}^n$ for every $0 \le k \le n$. (3 marks)

(b) Let X be a connected CW-complex. Then the inclusion induced homomorphism $\iota_\# : \pi_1(X^{(2)}) \to \pi_1(X)$ is an isomorphism. (3 marks)

(2) State and prove the Four lemma. (2+4 = 6 marks)

(3) Prove that the two definitions of the differential $\partial : C_q^{CW}(X) \to C_{q-1}^{CW}(X)$ of the cellular chain complex are the same. (6 marks)

(4) Compute the homology groups $H_n(X, A)$ when X is $\mathbb{S}^2$ or $\mathbb{S}^1 \times \mathbb{S}^1$ and A is a finite set of points in X. (3+3=6 marks)

(5) Use cellular homology to compute $H_*(\mathbb{S}^p \times \mathbb{S}^q)$. (6 marks)